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TS Model for Identification & Prediction of Non-linear System

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Abstract— *The main goal of this paper is to review the fuzzy systems of Takagi Sugeno (TS) for modelling a non-linear system. The Takagi-Sugeno models are a non-linear techniques, defined by a set of If- Then rules, each of which establishes a local linear input-output relationship between the variables of the model. The TS fuzzy model is trained using data obtained from the mathematical model of the non-linear system. The predicting results indicate that the Takagi-Sugeno fuzzy model gives a good accuracy. Simulations are done using MATLAB 2015a.*

Keywords*: Fuzzy modeling, Gradient descent, TS Model, fuzzy controller*

1. INTRODUCTION

Fuzzy Modeling based on the fuzzy set theory proposed by Zadeh [1] has been widely investigated in the literature. The aim of the whole exercise is to build fuzzy relations, which are expressed by a set of linguistic propositions derived, either from the experience of a skilled operator or a set of observed input-output data. In the early stages of knowledge based fuzzy logic applications to real life, Mamdani [2,3] has used Compositional Rule of Inference (CRI) form of fuzzy model to interpret the operator experience in handling simple operations. However, for some large complex systems, it is almost impossible to establish such knowledge based fuzzy models due to large number of fuzzy propositions and the highly complicated multidimensional fuzzy relationships. Later, the pioneering work of Takagi and Sugeno [4] on fuzzy modeling and control has led to several works in the literature [5,6] which are termed as multi model based approaches [7]. The basic idea in these approaches is to decompose the complicated input space into subspaces and then approximate the subsystem represented in each subspace by a linear regression model. Thus the overall fuzzy model is considered as a combination of interconnected subsystems with simpler models and was referred as TS-Model in the literature for fuzzy inference. Using the similar decomposition of the input space, CRI-model interpolates among parallel hyper surfaces perpendicular to the output co-ordinates resulting in a family of hyper surfaces depending upon the fuzziness around the parallel hyper surfaces. On the other hand, TS-model interpolates among the inclined hyper surfaces resulting in a single hyper surface. In the fuzzy modeling, the most important problem is the identification method of a system. The identification for the fuzzy modeling has two aspects: structure identification and parameter identification. Structure Identification of a fuzzy model consists of determining a suitable number and shape of fuzzy partitioning of input-output space, since the number of fuzzy partitions gives the number of rules and the shape of fuzzy partition determines the membership function parameters. Parameter identification involves fixing the parameters of premise and consequent in each rule.

In this paper, we have designed a fuzzy model for the identification and modeling of a non-linear system. A brief review of TS model is given in section II. Section III briefly reviews the gradient descent learning of the parameters of the fuzzy model. Simulation results on identification and modeling of a non-linear system are discussed in section IV followed by conclusions in section V.

2. FUZZY MODEL : TS-MODEL

Takagi-Sugeno (T-S) Model

The T-S model was proposed in an effort to develop a systematic approach of generating fuzzy rules from a given input-output data set. For this, TS model replaces the fuzzy sets in the consequent part (THEN) of the Mamdani rule with a linear equation of the input variables.

The T-S fuzzy model suggested in the year 1985 can represent or model a general class of static or dynamic non-linear systems. It is based on fuzzy partitioning of input space and can be viewed as the expansion of piecewise linear partition. This model thus, approximates a non-linear system with a combination of several linear systems by decomposing the whole input space into subspaces and representing each subspace with a linear equation. It can describe a highly non-linear system using a small number of rules. Moreover, due to the explicit functional representation form, it is convenient to identify its parameters using some learning algorithms. The inference performed by the T-S model is an interpolation of all the relevant linear models. The degree of relevance of a linear model is determined by the degree to which the input data belong to the fuzzy subspace associated with the linear model. These degrees of weight become the weight in the interpolation process. The identification of a T-S fuzzy model using input-output data consists of two parts: Structure identification (rule construction) and Parameters identification (fixing the parameters of premises and consequent in each rule). The consequent parameters are the coefficients of linear equations.

The fuzzy implications are formed by fuzzily partitioning the input space. The premise of a fuzzy implication determines a fuzzy subspace of the input space, the consequent of a fuzzy implication expresses a linear input-output regression relation which is valid in appropriate subspace. The T-S model is based on the idea to find a set of piece-wise linear structure to describe a non-linear relation. Each implication (rule) in the T-S model defines hyper plane in the premise-consequent product space. The overall output of the model is calculated by a weighted sum of each of the rule consequent.

Rules of T-S model are of the following form:

$$R^k : \text{ if } \; x^k \text{ is } A^k \text{ then } \; y \text{ is } f^k(x^k) \tag{1}$$

A linear form of $f^k(x^k)$ is as follows:

$$f^k(x^k) = b_{k0} + b_{k1} + b_{k2} + \dots\dots + b_{kn_k} x_{n_k} \tag{2}$$

where, $f^k(x^k)$ defines a locally valid model on the support of the cartesian product of fuzzy sets constituting the premise parts. The firing strength of each rule is calculated as follows:

The firing strength of the k^{th} rule, obtained by taking the T-norm (usually min or product operator) of the membership functions of the premise part of the rule is:

$$\mu^k(x^k) = \mu_1^k(x_1) \wedge \mu_2^k(x_2) \wedge \mu_3^k(x_3) \wedge \dots\dots \wedge \mu_{n_k}^k(x_{n_k}) \tag{3}$$

The normalized firing strength for the normalized calculation is then multiplied with output function $f^k(x^k)$. The normalized form of the overall output of the T-S model is defined as:

$$y^0 = \sum_{k=1}^{m} \frac{\mu^k(x^k)}{\sum_{j=1}^{m} \mu^j(x^j)} f^k(x^k) \tag{4}$$

and the non-normalized form of the overall output of the TS-model is defined as:

$$y^0 = \sum_{k=1}^{m} \mu^k(x^k) \; . \; f^k(x^k) \tag{5}$$

It is thus concluded that T-S model can express a highly non-linear functional relation using a small number of rules and its potential application is great.

3. GRADIENT DESCENT LEARNING

Fine Tuning of initial rules can be achieved by minimizing the objective function J. It is a function of square of error with respect to the parameters c_{ik}, a_{ik} and b_{ik}; a fuzzy model proposed by Takagi and Sugeno, is of the following form:

Rule i : IF x_1 *is* A_{i1} *and .....and* x_n *is* A_{in} THEN $y_i = c_{io} + c_{i1}x_1 + \dots\dots + c_{in}x_n$

(6)

where $i = 1,2,\dots\dots.l$, l is the number of IF-THEN rules, c_{ik} $(k = 0,1,2,\dots n)$ are consequents parameters. y_i is an output from the IF-THEN rule, and A_{ij} are the linguistic labels of fuzzy sets describing the qualitative nature of the input variable x_i.Given an input $(x_1, x_2, \dots, x_n)$, the final output of the fuzzy model is inferred as follows:

$$y = \sum_{i=1}^{l} w_i y_i \tag{7}$$

where, y_i is calculated for the input by the consequent equation of the i^{th} implication, and the weight w_i implies the overall truth value of the premise of the implication for the input, and calculated as:

$$w_i = \prod_{k=1}^{n} A_{ik}(x_k) \tag{8}$$

where

$A_{ik}(x_k) = \exp(-\frac{(x_k - a_{ik})^2}{b_{ik}^2})$ is the Gaussian membership function. Here, a_{ik} and b_{ik} are parameters of the membership functions. One may apply the gradient descent technique to modify the parameters a_{ik} , b_{ik} and c_{ik} .

From equations (6) - (8), the overall output is given as:

$$y = \sum_{k=0}^{n} \sum_{i=1}^{l} w_i c_{ik} x_k \qquad (9)$$

where, x_0 =1.

The performance of the model is measured by the following index :

$$E = \frac{1}{2}(y^* - y)^2 \qquad (10)$$

where, y and y^* denote the outputs of a fuzzy model and a real system, respectively. By partially differentiating E with respect to each parameter of a fuzzy model, we obtain:

$$\frac{\partial E}{\partial c_{ik}} = \frac{\partial E}{\partial y} \times \frac{\partial y}{\partial c_{ik}}$$

$$= -(y^* - y) w_i x_k = -\delta w_i x_k \qquad (11)$$

$$\frac{\partial E}{\partial a_{ik}} = \frac{\partial E}{\partial y} \times \frac{\partial y}{\partial a_{ik}}$$

$$= -(y^* - y) \frac{2(x_k - a_{ik})}{b_{ik}} w_i \sum_{k=0}^{n} c_{ik} x_k ,$$

$$= -\delta \frac{2(x_k - a_{ik})}{b_{ik}} w_i \sum_{k=0}^{n} c_{ik} x_k , \qquad (12)$$

$$\frac{\partial E}{\partial b_{ik}} = \frac{\partial E}{\partial y} . \frac{\partial y}{\partial b_{ik}}$$

$$= -(y^* - y) \frac{(x_k - a_{ik})^2}{b^2_{ik}} w_i \sum_{k=0}^{n} c_{ik} x_k ,$$

$$= -\delta \frac{2(x_k - a_{ik})^2}{b^2_{ik}} w_i \sum_{k=0}^{n} c_{ik} x_k \qquad (13)$$

where, $\delta = (y^* - y)$.

The final learning law can be defined as:

$$c_{ik}^{NEW} = c_{ik}^{OLD} + \varepsilon_1 \delta w_j x_k \qquad (14)$$

$$a_{ik}^{NEW} = a_{ik}^{OLD} + \varepsilon_2 \delta \frac{2(x_k - a_{ik}^{OLD})}{b_{ik}^{OLD}} w_j \sum_{k=0}^{n} c_{ik}^{OLD} x_k \qquad (15)$$

$$b_{ik}^{NEW} = b_{ik}^{OLD} + \varepsilon_3 \delta \frac{2(x_k - a_{ik}^{OLD})^2}{(b_{ik}^{OLD})^2} w_j \sum_{k=0}^{n} c_{ik}^{OLD} x_k \qquad (16)$$

where, $\varepsilon_1, \varepsilon_2$ and ε_3 are learning coefficients and $\varepsilon_1, \varepsilon_2$, ε_3 > 0. By using Eqns. (14-16), we can successively update the parameters, a_{ik} , b_{ik} and c_{ik} , until the value of the summation of δ for all data points is small enough.

4. SIMULATIONS AND RESULTS

Fuzzy model identification is implemented on a dynamic nonlinear plant described by the following equation:

$$y(k+1) = \frac{y(k).y(k-1)(y(k)+2.5}{1 + y^2(k) + y^2(k-1)} + p(k) \qquad (17)$$

where $p(k) = \sin(2\pi k/10)$

The parameters of the membership function (c_{ij}, σ_{ij}) and consequent parameters are randomly initialized and then gradient descent method is used for tuning of these parameters. We use the error function SE $= \frac{1}{2} e^2$, as a performance index (PI) of the fuzzy model, where $e = y^* - y$, y and y^* denote outputs of the fuzzy model and the real system, respectively. The approximate power of the identified model is then compared based on this performance index. Fig 1 shows the plot of the identified fuzzy model and fig 2 shows the plot of performance index.

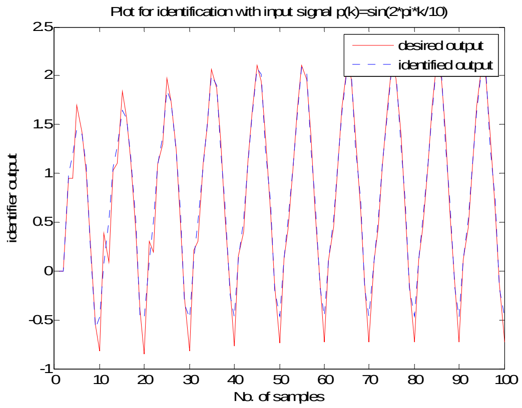

Fig 1: Plot of actual output vs. identified output

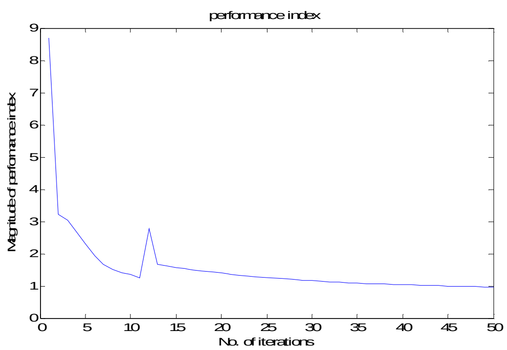

Fig 2: Performance index of identified model

After the model is identified, its parameters are fixed for the predication. Fig 3 shows a plot of desired output and the predicted output.

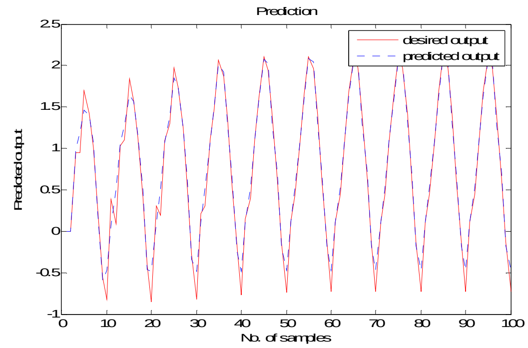

Fig 3: Plot of desired vs. predicted output

5. CONCLUSION

In this paper, we have presented a review of fuzzy modeling using TS model. First the TS model is designed for the non-linear system with gradient descent learning for the parameters of antecedent and consequent parameters. These parameters are then fixed and the prediction is done for the system.

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